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Bianchi Type-I Cosmological Model with a Varying Λ Term in Self Creation Theory of Gravitation

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Abstract Bianchi type-I cosmological model is investigated in the framework of Barber's second self creation theory of gravitation, with dust fluid as a source of the gravitational field, in presence of a non-zero time-dependent cosmological term. Various physical and geometrical aspects of the model are also discussed.

Keywords Anisotropic · Bianchi type-I model · Dust fluid · Self creation theory · Varying Λ term

1 Introduction

Scalar-tensor theories of gravity were first proposed by Jordan [8, 9], and then by Brans and Dicke [4] and Dicke [6, 7] as an alternative to Einstein's general theory of relativity. Later these theories were extended in a more general framework [2, 16, 31]. These represent a generalisation of the simplest scalar-tensor theory of gravity which is the Brans-Dicke theory [4]. The Brans-Dicke theory develops Mach's principle in a relativistic framework by assuming that inertial masses of fundamental particles are not constant, but are dependent upon the particles' interaction with some cosmic scalar field coupled to the large-scale distribution of matter in motion. Barber [1] has proposed two theories in 1982. The first one is

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a modified Brans and Dicke theory that is unsatisfactory since the equivalence principle is violated. The second one is an adaptation of general relativity to include continuous creation and is within the limits of the observations.

Anisotropic homogeneous universes play an important role in understanding some essential features of the universe, such as the formation of galaxies during its early stages of evolution. Various aspects of the self-creation theories have been investigated by Pimentel [17], Soleng [26, 27] and Reddy et al. [22]. Singh [24], Maharaj and Beesham [15] have obtained various properties of the two self-creation theories. Venkateswarlu and Reddy [28] have investigated Bianchi type-V radiating model in Barber's first theory. Venkateswarlu and Reddy [29] have analysed an anisotropic cosmological model, in self-creation theory, when the source of the gravitational field is a perfect fluid and the metric is of Marder's cylindrically symmetric form. Reddy [21] have investigated Bianchi type-I vacuum model in self-creation cosmology. Venkateswarlu and Reddy [30] have studied spatially-homogeneous and anisotropic Bianchi type-I models in Barber's second self-creation theory of gravitation for radiating and stiff-fluid cases. Recently Katore, Rane and Kurkure [11] have obtained plane symmetric cosmological models with negative constant deceleration parameter in self creation theory. Pradhan, Agarwal and Singh [20] have obtained singularity free cosmological solutions for an LRS Bianchi type-I metric in Barber's second self-creation theory of gravitation with perfect fluid for the time dependent deceleration parameter. Pradhan et al. [18, 19] have also studied self creation theory in different context.

In modern cosmological theories, a dynamic cosmological term $\Lambda(t)$ remains a focal point of interest as it solves the cosmological constant problem in a natural way. There are significant observational evidence for the detection of Einstein's cosmological constant, Λ or a component of material content of the universe that varies slowly with time and space to act like Λ. A wide range of observations now compellingly suggest that the universe possesses a non-zero cosmological term [13]. Scalar tensor theories with a dynamic cosmological term have been studied by Berman [3]. For some recent works on varying Λ cosmologies in scalar tensor theories the reader is advised to consult Berman [3] and references therein. Recently Singh and Chaube [25] have studied the effect of a time-dependent cosmological constant in a family of scalar tensor theories for Bianchi type I, III, V, VI_0 and Kantowski-Sachs models.

In this paper, we have investigated the spatially-homogeneous and anisotropic Bianchi type-I cosmological model with a varying Λ term for dust fluid in Barber's self-creation theory of gravitation. This paper is organized as follows: The metric and field equations are presented in Sect. 2. In Sect. 3, we deal with the solution of the field equations in presence of dust fluid as a source of the gravitational field. Section 4 includes various physical and geometrical features of the model. In Sect. 5, we have given the concluding remarks.

2 The Metric and Field Equations

We consider the Bianchi type I metric in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2, \tag{1}$$

where the metric potentials A, B, C are functions of time alone.

The energy momentum tensor has the form

$$T_i^j = (\varepsilon + p)v_i v^j + p g_i^j, \tag{2}$$

In (2), ε is energy density, p is the pressure and v^i is the four velocity vector satisfying the relation

$$g_{ij}v^i v^j = -1. \tag{3}$$

The Einstein's field equations in Barber's second self-creation theory are

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi\phi^{-1}T_i^j - \Lambda g_i^j, \tag{4}$$

and

$$\phi_{;k}^k = \frac{8\pi}{3}\eta T, \tag{5}$$

where $\phi_{;k}^k$ is the invariant d'Alembertian and the contracted tensor T is trace of energy momentum tensor describing all non-gravitational and non-scalar field matter and energy. Here η is a coupling constant to be determined from experiments. Because of the homogeneity condition imposed by the metric, the scalar field ϕ will be a function of t only.

We assume the coordinates to be comoving so that

$$v^1 = 0 = v^2 = v^3 \quad \text{and} \quad v^4 = 1. \tag{6}$$

For the line-element (1) the field equations (4) and (5) lead to the following system of equations:

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = -8\pi\phi^{-1}p - \Lambda, \tag{7}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = -8\pi\phi^{-1}p - \Lambda, \tag{8}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = -8\pi\phi^{-1}p - \Lambda, \tag{9}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{C_4 A_4}{CA} = 8\pi\phi^{-1}\varepsilon - \Lambda, \tag{10}$$

$$\phi_{44} + \phi_4\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = \frac{8\pi}{3}\eta(\varepsilon - 3p). \tag{11}$$

The energy conservation equation of general relativity $(T_i^j)_{;j} = 0$ takes the form

$$\varepsilon_4 + (\varepsilon + p)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0. \tag{12}$$

In general relativity, we define equivalent densities and pressures given by Soleng [26, 27] as

$$\varepsilon_{eq} = \frac{\varepsilon}{\phi}, \tag{13}$$

$$p_{eq} = \frac{p}{\phi}. \tag{14}$$

Using (13) and (14) in (12),we get

$$\left(\frac{\varepsilon}{\phi}\right)_4 + \left(\frac{\varepsilon + p}{\phi}\right)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0, \tag{15}$$

where the sub indice 4 in A, B, C denotes ordinary differentiation with respect to t. The velocity field v^i is irrotational. The scalar expansion θ and components of shear σ_{ij} are given by

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}, \tag{16}$$

$$\sigma_{11} = \frac{A^2}{3}\left[\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C}\right], \tag{17}$$

$$\sigma_{22} = \frac{B^2}{3}\left[\frac{2B_4}{B} - \frac{A_4}{A} - \frac{C_4}{C}\right], \tag{18}$$

$$\sigma_{33} = \frac{C^2}{3}\left[\frac{2C_4}{C} - \frac{A_4}{A} - \frac{B_4}{B}\right], \tag{19}$$

$$\sigma_{44} = 0. \tag{20}$$

Therefore

$$\sigma^2 = \frac{1}{3}\left[\frac{A_4^2}{A^2} + \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2} - \frac{A_4B_4}{AB} - \frac{B_4C_4}{BC} - \frac{C_4A_4}{CA}\right]. \tag{21}$$

3 Solutions of the Field Equations

Dust-fluid corresponds to the equation $p = 0$. For this case, (15) on integration leads to

$$\left(\frac{\varepsilon}{\phi}\right) ABC = \alpha, \tag{22}$$

where α is a constant of integration.

Now adding (7), (8) and (9),we get

$$2\frac{A_{44}}{A} + 2\frac{B_{44}}{B} + 2\frac{C_{44}}{C} + \frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{C_4A_4}{CA} = -24\pi\phi^{-1}p - 3\Lambda. \tag{23}$$

Adding three times of (10) in (23), we obtain

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + 2\frac{A_4B_4}{AB} + 2\frac{B_4C_4}{BC} + 2\frac{C_4A_4}{CA} = -12\pi\phi^{-1}(-\varepsilon + p) - 3\Lambda. \tag{24}$$

For dust fluid case ($p = 0$), the (24) reduces to

$$\frac{(ABC)_{44}}{ABC} = 12\pi\left(\frac{\varepsilon}{\phi}\right) - 3\Lambda. \tag{25}$$

Using (22) and (25), we get

$$(ABC)_{44} = 12\pi\alpha - (3\Lambda)ABC. \tag{26}$$

Now integrating (26) with the assumption that $\Lambda = \frac{k_8}{ABC}$, where k_8 is a constant, we obtain

$$ABC = (c_1t + c_2)^2, \tag{27}$$

where $c_1^2 = 6\pi\alpha - \frac{3}{2}k_8$, $2c_1c_2 = k_9$, $c_2^2 = k_{10}$, and k_9, k_{10} are constants of integration.

Hence (27) gives the values of metric potentials as,

$$A = A_0(c_1t + c_2)^{p_1}, \tag{28}$$

$$B = B_0(c_1t + c_2)^{p_2}, \tag{29}$$

$$C = C_0(c_1t + c_2)^{p_3}, \tag{30}$$

where the constants p_1, p_2, p_3 and A_0, B_0, C_0 satisfy the relations

$$p_1 + p_2 + p_3 = 2 \quad \text{and} \quad A_0B_0C_0 = 1. \tag{31}$$

Using (22) in (11), we obtain

$$(ABC\phi_4)_4 = \frac{8\pi}{3}\eta\alpha\phi, \tag{32}$$

now particularly taking $\alpha = 0$ we get

$$(ABC\phi_4)_4 = 0, \tag{33}$$

which on integration leads to

$$\phi = k_4 - \frac{k_3}{c_1}(c_1t + c_2)^{-1}, \tag{34}$$

where k_3 and k_4 are constants of integration.

Hence the metric (1) leads to

$$ds^2 = -dt^2 + A_0^2(c_1t + c_2)^{2p_1}dx^2 + B_0^2(c_1t + c_2)^{2p_2}dy^2 + C_0^2(c_1t + c_2)^{2p_3}dz^2. \tag{35}$$

After suitable transformation of co-ordinates the metric (35) reduces to the form

$$ds^2 = -\frac{1}{c_1^2}dT^2 + T^{2p_1}dX^2 + T^{2p_2}dY^2 + T^{2p_3}dZ^2. \tag{36}$$

Equation (36) represents the Bianchi type-I, spatially-homogeneous and anisotropic model with dust fluid in Barber's self-creation theory of gravitation.

4 Some Physical and Geometrical Features

The energy density (ε) for the model (36) is given by

$$\varepsilon = \frac{k_5}{T^2}\left(k_2 - \frac{k_6}{T}\right), \tag{37}$$

where the constants k_5 and k_6 are given by

$$k_5 = \frac{(p_1p_2 + p_2p_3 + p_3p_1)c_1^2}{8\pi}, \tag{38}$$

$$k_6 = \frac{k_1}{c_1}. \tag{39}$$

The reality condition $\varepsilon > 0$ leads to $T > \frac{k_6}{k_2}$ and $k_5 > 0$.

The scalar field (ϕ) is given by

$$\phi = \left(k_4 - \frac{k_3}{c_1 T}\right), \tag{40}$$

which is not defined at $T = 0$.

The expansion (θ) and shear (σ) are given by

$$\theta = \frac{2c_1}{T}, \tag{41}$$

$$\sigma_1^1 = \frac{c_1}{3}(2p_1 - p_2 - p_3)\frac{1}{T}, \tag{42}$$

$$\sigma_2^2 = \frac{c_1}{3}(-p_1 + 2p_2 - p_3)\frac{1}{T}, \tag{43}$$

$$\sigma_3^3 = \frac{c_1}{3}(-p_1 - p_2 + 2p_3)\frac{1}{T}, \tag{44}$$

$$\sigma_4^4 = 0. \tag{45}$$

Hence

$$\sigma^2 = \frac{k_7}{T^2}, \tag{46}$$

where $k_7 = \frac{{c_1}^2}{3}[{p_1}^2 + {p_2}^2 + {p_3}^2 - p_1 p_2 - p_2 p_3 - p_3 p_1]$.

It is worth mentioning that $\frac{\varepsilon}{\theta^2} = \frac{k_5}{4c_1^2}(k_2 - \frac{k_6}{T})$. Hence if $k_6 \to 0$, i.e. $k_1 \to 0$ or $T \to \infty$ then $\frac{\varepsilon}{\theta^2}$ becomes constant, which means that the energy density (ε) is proportional to the square of the scalar expansion (θ^2). Interestingly we have $\lim_{t\to 0}(\frac{\varepsilon}{\theta^2}) = \text{constant}$. Thus the model approaches homogeneity and matter is dynamically negligible near the origin. This behaviour of the model is similar to the results given by Collins [5].

The spatial volume (V) is given by

$$V = R^3 = \sqrt{(-g)}ABC = T^2. \tag{47}$$

The cosmological constant (Λ) is given by

$$\Lambda = \frac{k_8}{T^2}, \tag{48}$$

where $k_8 = [p_1(1 - p_1) + p_3(1 - p_3) - p_1 p_3]{c_1}^2$.

The deceleration parameter (q) is given by

$$q = -\frac{R_{44}R}{{R_4}^2} = \frac{1}{2c_1}, \tag{49}$$

where R is the scale factor given by (47).

If the deceleration parameter (q) is positive our model (36) decelerates and for q to be negative the model inflates. Clearly $T \to 0$ gives $\Lambda \to \infty$ and $T \to \infty$ gives $\Lambda \to 0$. When T is finite Λ becomes a constant. It is worth showing that Λ is inversely proportional to the square of time T. The value of cosmological constant Λ is in an excellent agreement with observations [12, 23] of type Ia Supernovae (SNe). The main conclusion of these observations is that the expansion of the universe is accelerating and the cosmological term was very large at initial times which relaxes to a genuine cosmological constant with due course of time.

The three components of Huble parameter are given by

$$H_1 = \frac{A_4}{A} = \frac{p_1 c_1}{T}, \tag{50}$$

$$H_2 = \frac{B_4}{B} = \frac{p_2 c_1}{T}, \tag{51}$$

$$H_3 = \frac{C_4}{C} = \frac{p_3 c_1}{T}. \tag{52}$$

Hence the Hubble parameter H is given by

$$H = \frac{2c_1}{3T}. \tag{53}$$

The anisotropy parameter $\overline{A}$ is defined by

$$\overline{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \tag{54}$$

where $\Delta H_i = H_i - H \ (i = 1, 2, 3)$.

For our model the anisotropy parameter $\overline{A}$ is given by

$$\overline{A} = \frac{c_1}{3T}\left(\frac{k_9}{T} - 4\right), \tag{55}$$

where $k_9 = c_1(p_1^2 + p_2^2 + p_3^2)$.

The expansion velocity R_4 is given by

$$R_4 = \frac{2}{3}(c_1 t + c_2)^{-1/3} = \frac{2}{3}T^{-1/3}. \tag{56}$$

The expansion velocity R_4 diverges as $t \to -\frac{c_2}{c_1}$, i.e. $T \to 0$. Hence the expansion of the universe is infinite as we approach towards $t \to -\frac{c_2}{c_1}$.

It is evident from the above physical quantities that the fluid has non-zero expansion (i.e. $\theta \neq 0$). It is also observed that when $T \to 0$ i.e., $t \to -\frac{c_2}{c_1}$, the physical quantities for the model (35) and (36), like expansion scalar (θ), shear scalar (σ), cosmological constant (Λ) and the Hubble parameter (H) diverge.

Also the spatial volume V is zero at $T = 0$ i.e., $t = -\frac{c_2}{c_1}$. The expansion scalar θ is infinite at initial singularity $T = 0$, which shows that the universe starts evolving with zero volume and infinite rate of expansion at $T = 0$. Initially at $T = 0$ the energy density ε is infinite. The anisotropy parameter $\overline{A}$ and shear scalar σ also tend to infinity at the initial epoch $T = 0$ and both vanishes at $T \to \infty$.

As T increases, the spatial volume V increases but the expansion scalar θ decreases, thus the expansion rate decreases as time increases. As $T \to \infty$ the spatial volume V becomes infinitely large and the expansion in the model stops. All the physical parameters ε, p, σ, H_1, H_2 and H_3 tend to zero when $T \to \infty$. Therefore, the model (36) essentially gives an empty universe for large values of T. All the physical quantities remain finite and physically significant at finite region of the universe. Also clearly the scalar field remains finite through the evolution of universe. In case $\eta \to 0$, the solutions approach Einstein's general theory of relativity in all respect and the model represents shearing, non-rotating and expanding universe with a big bang start.

The space-time exhibits POINT TYPE singularity at $T = 0$, i.e. $t = -\frac{c_2}{c_1}$ with $p_1 > 0$, $p_2 > 0$, and $p_3 > 0$. When $T = 0$ with either $p_1 > 0$, $p_2 > 0$, $p_3 < 0$ or $p_1 < 0$, $p_2 > 0$, $p_3 > 0$ or $p_1 > 0$, $p_2 < 0$, $p_3 > 0$, then there is a CIGAR TYPE singularity in the model [14].

For the model (36), the particle horizon exists because

$$\int_{T_0}^{T} \frac{dt}{R(t)} = \int_{T_0}^{T} \frac{dt}{(c_1 t + c_2)^{2/3}} = \left[\frac{3}{c_1}(c_1 t + c_2)^{1/3}\right]_{T_0}^{T} \tag{57}$$

is a convergent integral.

5 Concluding Remarks

The models obtained, in this paper, are of considerable interest and may be useful in self-creation cosmology to study the large scale dynamics of the physical universe. From (41), we find that the model will represent an expanding universe. However, the expansion in the model $\theta \to 0$ when $T \to \infty$. Thus the expansion in the model decreases as time increases. It is also observed that the model is shearing, non-rotating and non-singular. However, the scalar field in the model possesses an initial singularity.

Since $\lim_{T\to\infty}(\frac{\sigma}{\theta}) = \frac{k_7}{2c_1}$, which is a non-zero constant, hence the model does not approach isotropy for large values of T in general, but if $k_7 = 0$, then the model approaches isotropy.

The anisotropic expansion of the universe with time is evident from the model (36). When $k_3 = 0$, the scalar field ϕ is constant and in this case the model (36) represents the Bianchi type-I model with a variable cosmological term in the Einstein's general theory of relativity. The value of cosmological constant Λ is in an excellent agreement with observations [12, 23] of type Ia Supernovae (SNe). The main conclusion of these observations is that the expansion of the universe is accelerating and the cosmological term was very large at initial times which relaxes to a genuine cosmological constant with due course of time. Our model (36) takes the form of the well known Kasner's universe [10]. Hence it is a generalization of the Kasner's model in self creation cosmology proposed by Barber [1].

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